

# Revision 4

Semester Two Examination

Question/Answer Booklet

## MATHEMATICS METHODS UNITS 1 AND 2

### Section Two: Calculator-assumed

Student Number: In figures

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In words

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Teacher name

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### Materials required/recommended for this section

#### *To be provided by the supervisor*

This Question/Answer Booklet

Formula Sheet (retained from Section One)

#### *To be provided by the candidate*

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

### Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

**Section Two: Calculator-assumed****65% (66 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 65 minutes.

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**Question 1****(7 marks)**

(a) A sequence is defined by  $T_{n+1} = T_n - 7$ ,  $T_1 = 111$ .

(i) Determine  $T_{20}$ . (1 mark)

(ii) The sum of the first 40 terms,  $S_{40}$ . (1 mark)

(iii) The value of  $n$  that maximises  $S_n$ . (2 marks)

(b) A geometric sequence with  $T_2 = 87.5$  has a sum to infinity of 800. Determine all possible values of  $T_1$  for this sequence. (3 marks)

**Question 2****(8 marks)**

- (a) Two students are to be chosen from a class of 18.
- (i) Determine how many different pairs of students may be chosen. (1 mark)
- (ii) One of the students in the class is the oldest in the school. What is the probability that this student is included in the pair chosen? (2 marks)
- (b) A box contains 13 cans of soup, four of which have tomato as an ingredient and the remainder that do not. Four cans are to be selected at random from the box.
- (i) Calculate how many different selections of four cans can be made from the box. (1 mark)
- (ii) Determine the probability that none of the four cans will have tomato as an ingredient. (2 marks)
- (iii) Determine the probability that in the selection of four cans, there will be an equal number of cans with and without tomato as an ingredient. (2 marks)

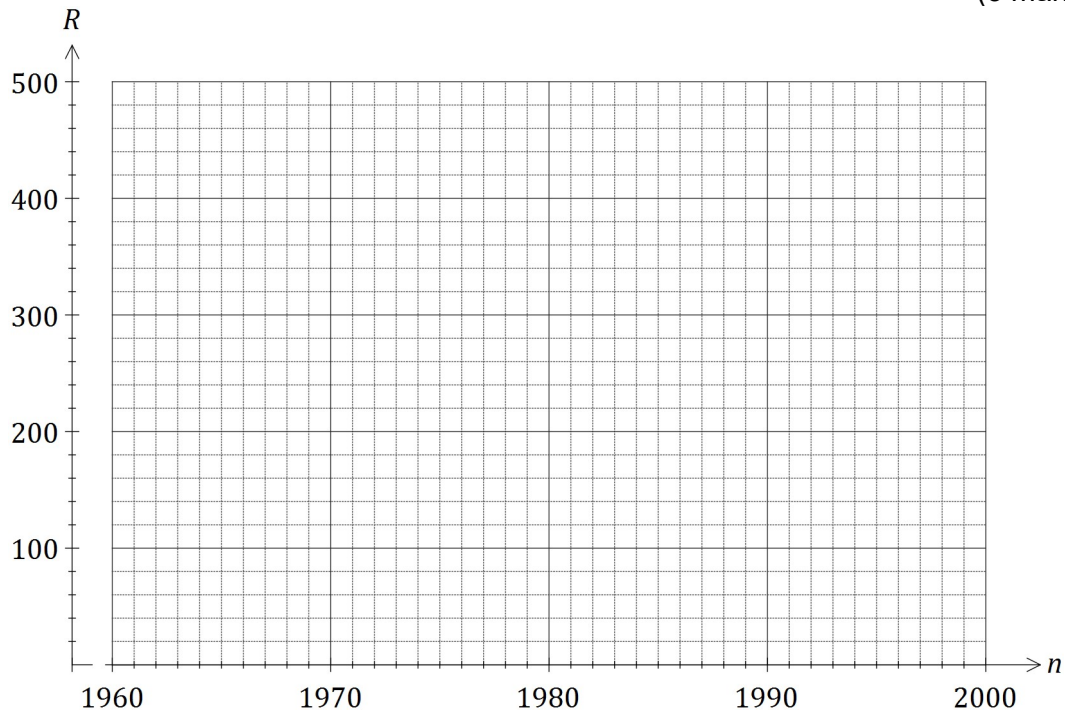
**Question 3****(8 marks)**

The imprisonment rate  $R$ , in number of prisoners per 100 000 people, in the US between the years 1960 and 2000, can be modelled by the following equation, where  $n$  is the year.

$$R = 85(1.038)^{n-1960}$$

(a) Calculate the imprisonment rate in the year 2000. (1 mark)

(b) Draw the graph of the imprisonment rate for  $1960 \leq n \leq 2000$  on the axes below. (3 marks)



(c) The population of the US was 266 million in 1995. Determine the number of prisoners in the US at this time, to the nearest 1 000. (3 marks)

(d) When  $R$  first exceeded 500, steps were taken to address the exponential growth in the prison population and the model no longer applied. In what year did this occur? (1 mark)

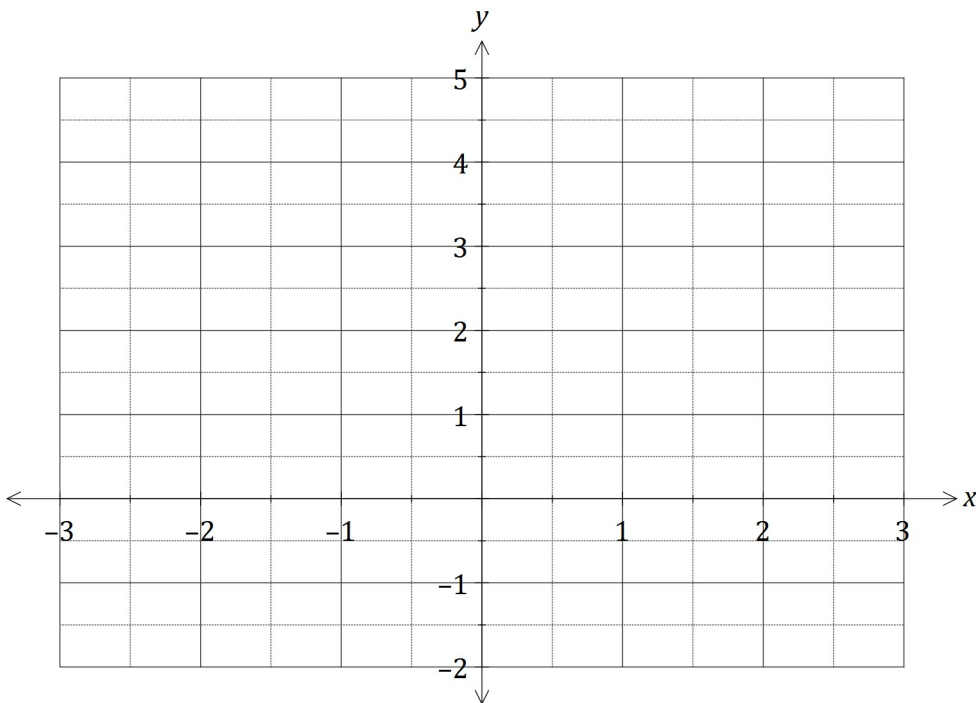
**Question 4****(9 marks)**

The function  $f$  is given by  $f(x) = x^3 - 3x + 2$ .

(a) Show that the graph of  $y = f(x)$  has two roots and state their coordinates. (2 marks)

(b) Use calculus techniques to determine the coordinates of all stationary points of the graph of  $y = f(x)$ . (4 marks)

(c) Sketch the graph of  $y = f(x)$  on the axes below for  $-2 \leq x \leq 2$ . (3 marks)

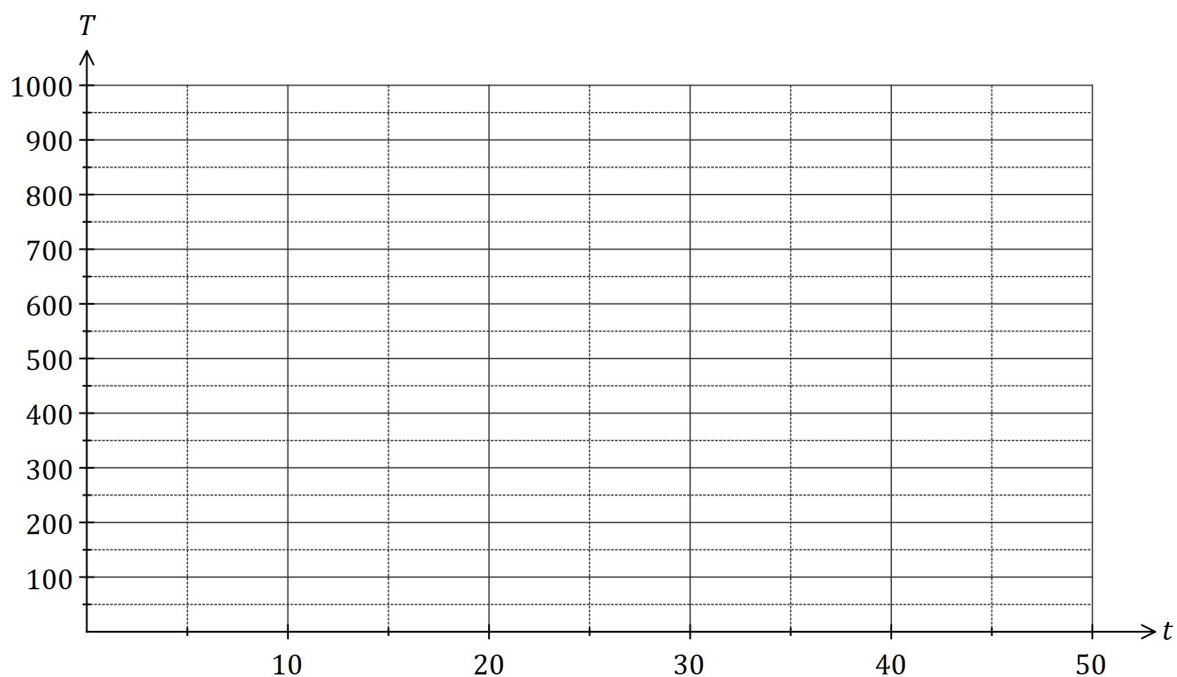


**Question 5****(9 marks)**

The temperature  $T$  of a cast taken out of an oven cools according to the model  $T = 960(0.93)^t$ , where  $t$  is the time in minutes since the cast was removed from the oven.  $T$  is measured in  $^{\circ}\text{C}$ .

- (a) Determine the fall in temperature of the cast during the first 3 minutes. (2 marks)

- (b) Graph the temperature of the cast against time on the axes below. (4 marks)



- (c) State the name of this type of function. (1 mark)

- (d) The temperature of the cast falls to room temperature of  $14^{\circ}\text{C}$ .

- (i) Determine the time taken for the cast to reach room temperature. (1 mark)

- (ii) Comment on the usefulness of the model for large values of  $t$ . (1 mark)

**Question 6****(8 marks)**

- (a) Calculate the area of the minor segment that subtends an arc of  $150^\circ$  in a circle of diameter 190 cm. (2 marks)
- (b) A chord of length 33 cm subtends an angle of  $\frac{\pi}{11}$  at the centre of a circle. Calculate the radius of the circle. (2 marks)
- (c) Parallelogram  $PQRS$  has side  $PQ = 35$  cm, side  $QR = 18$  cm and an area of  $200$  cm<sup>2</sup>. Determine the lengths of the diagonals of  $PQRS$ . (4 marks)

**Question 7****(8 marks)**

A council took a random sample of 125 and 172 properties from suburbs  $P$  and  $Q$  respectively. A total of 36 of the properties in the sample were in arrears with their rates, and 21 of these properties were in suburb  $Q$ . 'In arrears' means that payment of rates is overdue.

- (a) Council officers wanted to choose 4 of the properties that were in arrears. How many different selections of properties are possible? (2 marks)
- (b) Determine the probability that one randomly chosen property from the sample
- (i) is not in arrears and is in suburb  $Q$ . (2 marks)
- (ii) is in suburb  $P$  given that it is in arrears. (1 mark)
- (c) Justifying your answer with conditional probabilities, comment on whether being in arrears with rates is independent of the suburb the property is in. (3 marks)



**Question 8****(8 marks)**

A pyramid with a rectangular base of length  $L$  and width  $w$  has perpendicular height  $h$ . The length of the base is five times its width and the sum of the width, length and height is 117 cm.

- (a) Calculate the length, height and volume of the pyramid when  $w = 15$  cm. (2 marks)
- (b) Show that the volume of the pyramid is given by  $V = 195w^2 - 10w^3$ . (2 marks)
- (c) Use calculus to determine the maximum volume of the pyramid and state the dimensions required to achieve this volume. (4 marks)